Finite Pointset Method for Low-Mach-Number Flows

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In the past three years, the so-called Finite Pointset Method (FPM), a new powerful meshfree method for simulations in incompressible and compressible fluid dynamics, was developed at the ITWM (Institute for industrial mathematics), Kaiserslautern, Germany. FPM can be considered as a general finite difference method for the Navier-Stokes equations, including also heat transport phenomena. The numerical scheme of FPM is based on the classical projection idea of Chorin, however this idea was further developed for the purpose of solving compressible phenomena. Moreover, for establishing generalized finite differences, some Moving Least Squares (MLS) approach is employed.

The developments concerning FPM are comprised of two parts: 1.) Setting up a projection method that works properly for incompressible as well as compressible phenomena, 2.) Making these ideas applicable for the meshfree framework within FPM. These two groups of tasks and ways of their solutions will be mentioned in the presentation, and are briefly sketched below

1.) FPM is based on moving points, i.e. numerical points which are not geometrically connected by any mesh. Neighbourhood relations are simply given by the geodesic distance. FPM is a Lagrangian method, i.e. the points move with the velocity of the continuum. Analytical derivatives of specially constructed MLS approximations are used in order to approximate the spatial derivatives in the partial differential equations. The employment of these analytical derivatives guarantees certain continuous conservation properties. The scheme for incompressible as well as compressible fluid flows is based, as already mentioned, on Chorin's projection idea. In a first step, a forward approximation of the velocity at the time level n+1 is computed by directly approximating the Navier-Stokes-equations using the present flow quantities at time level n; the energy equation is solved in the same explicit sense. The second step is the correcting step: here the velocity field and the pressure field are corrected such that finally the changes of velocity, pressure and temperature can be viewed as the results of an implicit time step under certain compressibility assumptions. The compressibility of the fluid must be expressed by an equation of state where the density is a function of pressure and temperature. The correction step leads to a Helmholtz-type differential equation.

2.) The extension of Chorins projection method leads, as mentioned, to a Helmholtz-type PDE, which needs to be solved on the set of numerical points, which do not posses a geometrical link by some mesh. One requirement is the very precise approximation of second-order spatial derivatives within these point clouds. We will present a very efficient method of third order. A second requirement is, out of this, the constitution of a big, but sparse linear system. Here, we will speak about the involvement of certain stability criteria. The third task is the efficient solution of the linear system mentioned. Several ideas, including multilevel approaches, will be presented and their efficiency will be demonstrated.